JET CONFIGURATION IN ACCOMPANYING FLOWS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 2, pp. 250–257, 1966

UDC 536.46

A method is described for integrating on a MN-7 analog computer the energy equations relevant to a moving medium with heat sources, for the two-dimensional adiabatic problem, allowing for compressibility. Jet temperature and velocity fields for accompanying flows in the combustion of methane, obtained by machine integration, are presented.

The temperature fields of a burning jet are determined by the complex interaction of heat and mass transfer processes in compressible turbulent jets in the presence of internal heat sources. At the present time we cannot obtain an exact analytical solution of the system of equations describing this process.

A description is given in [1] of a method of approximate integration on an analog computer of the heat conduction equation with internal heat sources, to determine the turbulent flame propagation velocity in the conditions of the one-dimensional problem.

In that paper an attempt was made to determine the temperature field of the flame formed by a homogeneous fuel-air jet of finite thickness flowing in an infinite accompanying stream of furnace gases. The problem has been solved for the case of a compressible medium, using the approximation hypothesis of [2].

It was shown by Vulis [4] that on transition to the generalized variables $U=\sqrt{\rho}u$, $V=\sqrt{\rho}v$, $J=\sqrt{\rho}c_p\Delta T$ the equations of energy of motion, diffusion, and continuity for compressible and incompressible media become identical. This conclusion is approximate, however, since in the transformation the continuity equation is written in the form

$$\sqrt{\rho}\frac{\partial U}{\partial x}+\sqrt{\rho}\frac{\partial V}{\partial y}+U\frac{\partial\sqrt{\rho}}{\partial x}+V\frac{\partial\sqrt{\rho}}{\partial y}=0, \quad (1)$$

and for the concept of identity we must take

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0. \quad (2)$$

Numerous experimental data of a number of authors confirm the basic position of the hypothesis that $\rho u^2 =$ = idem for compressible and incompressible flows, and the validity is therefore confirmed of replacing (1) by (2) for technical calculations. Hence an important conclusion follows regarding the possibility of using solutions obtained in conditions of incompressibility for problems where compressibility of the medium is taken into account in describing the processes in the generalized variables. We give below an approximate system of Eqs. (1) in generalized coordinates, describing combustion in moving compressible and incompressible streams, written in terms of mean values, using the hypothesis that

$$\frac{\partial(-\overline{U'V'})}{\partial y}=\varepsilon_v\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial x^2}\right),$$

$$\frac{\partial(-\overline{U'J'})}{\partial y}=\varepsilon_T\left(\frac{\partial^2 J}{\partial y^2}+\frac{\partial^2 J}{\partial x^2}\right).$$

This system has the form

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\varepsilon_v\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial x^2}\right), \quad (3)$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0, \quad (4)$$

$$U\frac{\partial J}{\partial x}+V\frac{\partial J}{\partial y}= =\varepsilon_T\left(\frac{\partial^2 J}{\partial y^2}+\frac{\partial^2 J}{\partial x^2}\right)+qW(C,J), \quad (5)$$

$$U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y}= =D\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial x^2}\right)-W(C,J), \quad (6)$$

where $U=\sqrt{\rho}u$, $V=\sqrt{\rho}v$, $J=\sqrt{\rho}c_p\Delta T$ and $C=\sqrt{\rho}\Delta C$. We will call this system (I).

It is known that in the conditions of the adiabatic problem (i + chemical energy = const), system (I) may be replaced by the single Eq. (5), with the known functions U = U(x,y) and V = V(x,y), which are determined theoretically for incompressible media or are found on the basis of correlation of test data. We have assumed that the Vulis hypothesis is applicable to turbulent jets in the presence of heat sources, i.e., to flames.

In solving the energy Eq. (5) in order to find the quantity J, we made use of the scheme of the asymptotic boundary layer

$$\varepsilon_T = kb_T(U_{max}-U_{min}),$$

and the Prandtl number was taken equal to 0.75, i.e.,

$$\sigma=\varepsilon_v/\varepsilon_T=0.75.$$

The energy equation is a nonlinear second-order partial differential equation of elliptic type with two independent variables. The coefficients U and V are functions of x and y. The equation has also the nonlinear term qW(J).

The MN-7 analog computer does not permit simulation of an equation of this type, and we therefore had to use an artificial method of solution. The most tenable in this case is the method of lines [3], which allows us to reduce the nonlinear partial differential equation to a system of ordinary differential equations in which the desired quantity J is a function of only one independent variable y, while the coefficients U_j, V_j of the system are also functions of y for each fixed value of the coordinate x. The number of ordinary differential equations in the system depends on the step, i.e., the chosen degree of accuracy of the solution. Thus, integration of the original energy equation is reduced to integration of a system of differential equations, which we will call system (II):

$$U_1 \frac{J_1 - J_{\text{in}}}{\Delta h} + V_1 \frac{dJ_1}{dy} = \varepsilon_{T_1}\left(\frac{d^2 J_1}{dy^2} + \frac{J_{\text{in}} - 2J_1 + J_2}{\Delta h^2}\right) + qW(J_1),$$

$$U_2 \frac{J_2 - J_1}{\Delta h} + V_2 \frac{dJ_2}{dy} = \varepsilon_{T_2}\left(\frac{d^2 J_2}{dy^2} + \frac{J_1 - 2J_2 + J_3}{\Delta h^2}\right) + qW(J_2),$$

. . . .

$$U_j \frac{J_j - J_{j-1}}{\Delta h} + V_j \frac{dJ_j}{dy} = \varepsilon_{T_j}\left(\frac{d^2 J_j}{dy^2} + \frac{J_{j-1} - 2J_j + J_{j+1}}{\Delta h^2}\right) + qW(J_j),$$

$$j = 1, 2, 3, 4, \ldots, n$$

with the boundary conditions

$$J = J_{\text{in}} \text{ when } x = x_{\text{in}},$$
$$J_1 = J_1^0 \text{ when } y = y_1,$$
$$J_2 = J_2^0 \text{ when } y = y_2,$$
$$J_j = J_j^0 \text{ when } y = y_j.$$

The set of solutions of the system of equations comprises the solution of the original energy equation.

The analog computer does not permit us to find the solution of the system of equations (II) in the general form, but it does allow us to find the temperature distribution in a two-dimensional space with specific values of the system coefficients and boundary conditions.

Results are given below of the integration of system (II) in order to determine the configuration of the flame for a jet of finite width $2b_0 = 60$ mm of a uniform mixture of methane and air (air-fuel ratio $\alpha = 1.87$) with initial temperature $T_0 = 473°$ K and velocity $u_1 = 46.5$ m/sec into an infinite accompanying stream of furnace gases having a velocity of $u_2 = 22$ m/sec and a temperature $T_{\max} = 1673°$ K, equal to the theoretical temperature for combustion of the mixture (adiabatic problem).

Using the Vulis hypothesis, in addition to the assumptions indicated above, we will calculate the aerodynamic pattern. The width of the dynamic boundary layer in the initial and main sections was determined by Abramovich's wake formulas [6] with m < < 0.35 ($m = U_2/U_1$) and coefficient of initial nonuniformity equal to 1.

For the initial section

$$b_v = c_{\text{in}} x (1 - m)/(1 + m), \text{ where } c_{\text{in}} = 0.27.$$

In the main section

$$b_v = 0.22x.$$

The width of the thermal layer in the initial and main sections was assumed from the condition that the Prandtl number $\sigma = 0.75$. The length of the initial dynamic section, and the attenuation of the value of U along the axis of the flow were calculated from the formulas [4]

$$x_{\text{in}}^{\text{d}} = \frac{1}{c_{\text{in}}(0.416 + 0.134m)},$$

$$\Delta U_m = \sqrt{\frac{1}{c_{\text{main}}(x - x_0)} \frac{\Phi(\xi)}{A_1(1 - m)}},$$

where

$$\Phi(\xi) = \frac{2 + 2.43\xi + \xi^2}{\xi + \xi^2} - \frac{2}{\xi^2} \ln(1 + \xi),$$

$$\xi = \mu/\alpha \Delta U_m, \quad \mu = m/(1 - m).$$

For simplicity the transition section was excluded from examination. The nonlinear coefficients U_j, V_j of the system in the dynamic boundary layer were computed analytically according to the relations obtained by Vulis et al. [4] in solving the equations of motion and continuity for an incompressible fluid within the limits of the asymptotic layer for accompanying flows,

$$\frac{U}{U_1} = m + \frac{1}{2}(1 + m)\{1 + \Phi(\varphi + 0.33)\},$$

$$V = \frac{1}{2} aU_1(m - 1)\left\{\int_0^{\varphi + 0.33} \Phi(z)\,dz - \varphi \Phi(\varphi + 0.33) + 0.364\right\}$$

where $\Phi(z)$ is the error integral, $\varphi = y/ax$.

The heat liberated from the chemical reaction was determined from the equation

$$qW(T) = qk_0 \beta C_{\text{CH}_4}^{\text{N}} C_{\text{O}_2}^{\text{N}} \exp\left(-\frac{E}{RT}\right)\left(\frac{273}{T}\right)^2.$$

Here

$$C_{\text{CH}_4}^{\text{N}} = C_{\text{CH}_4}^{0} \frac{T_{\max} - T}{T_{\max} - T_0},$$

and the normal concentration of oxygen $C_{\text{O}_2}^{\text{N}}$ was found from stoichiometric relations. The remaining quantities in the equation were taken from the data of [7]. The transition to the function relation was accomplished with the aid of the implicit connection between the temperature T and the excess enthalpy flux J. Before integration the system of differential equations

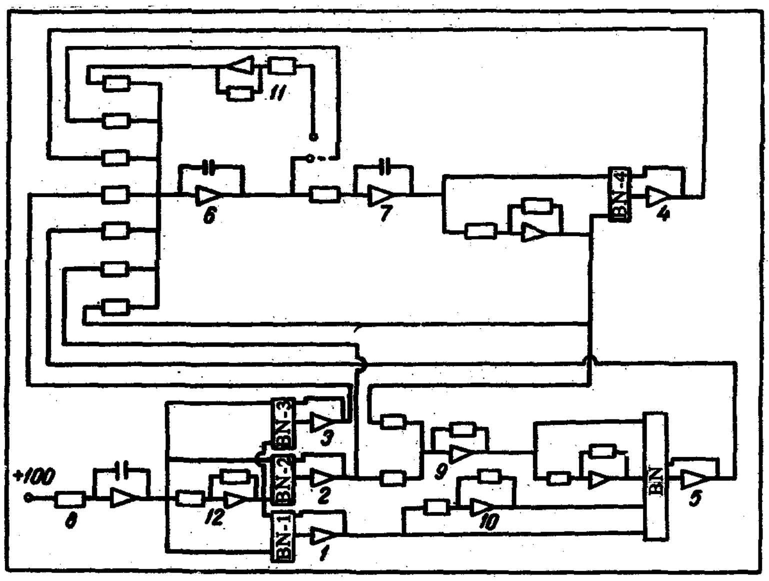

Fig. 1. Block diagram of the set-up for solving the equations of system (II) on the MN-7 machine: 1, 2, 3, 4) nonlinearity units for reproducing the nonlinear functions U_j, J_{j-1}, J_{j-2}, $qW(J_j)$; 5) product $U_j(J_{j-1} - J_j)$ unit; 6, 7, 8) d^2J_j/dy^2, dJ/dy, dy/dt integrators, respectively; 9) adder; 10, 11, 12) invertors.

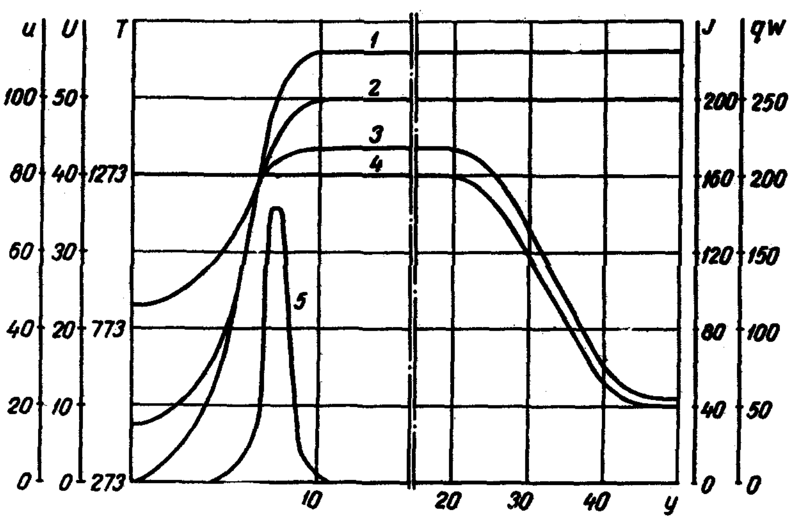

Fig. 2. Variation of the flow parameters in the cross section of the boundary layer of the initial thermal section $X_N^T = 0.236$ m at $\bar{x} = 7.87$ (y in mm): 1) temperature T (°K); 2) excess enthalpy $J = (\rho)^{1/2} c_p \Delta T$ ($j/m^{3/2} \cdot kg^{1/2}$); 3) longitudinal component of effective velocity u (m/sec); 4) the group $V = (\rho)^{1/2} u$ ($kg^{1/2}/m^{1/2} \cdot sec$); 5) amount of heat generated by the heat sources qW(J) ($MJ/m^3 \cdot sec$).

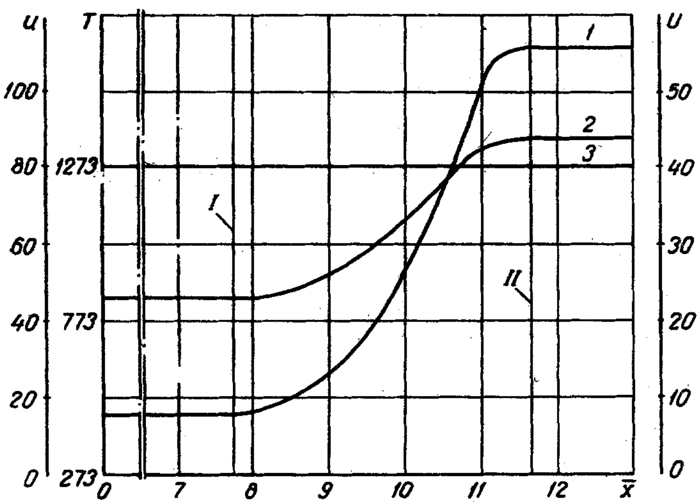

Fig. 3. Variation of the stream parameters along the flame axis: 1) temperature T (°K); 2) longitudinal component of effective velocity u (m/sec); 3) the group $V = (\rho)^{1/2}u$ ($kg^{1/2}/m^{1/2} \cdot sec$); I) boundary of the initial thermal section; II) boundary of the flame.

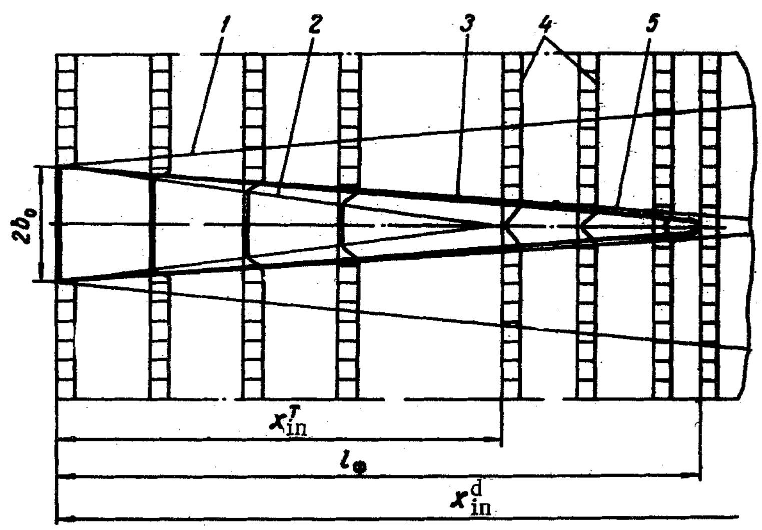

Fig. 4. Configuration of the flame, temperature fields in accompanying flows, and edges of the boundary layer in generalized coordinates: 1) outer edge of the thermal and dynamic boundary layer; 2) inner edge of the thermal boundary layer; 3) inner edge of the dynamic boundary layer; 4) temperature curves; 5) configuration of flame.

(II) was somewhat transformed. Thus, due to considerations of the method and the peculiarities of operation of the machine, we changed the direction of the axes x, y, and all the equations of the system were solved for the second derivative. In addition, for simplicity of solution the term $(J_{j-1}-2J_j+J_{j+1})/\Delta h^2$ was replaced, at the expense of some inaccuracy, by $(J_{j-2}-2j_{j-1}+J_j)/\Delta h^2$.

The boundary conditions of the initial and main thermal sections were examined separately. For the initial section we took the initial conditions to be the values of J(T) at the outer and inner edges of the thermal boundary layer:

$$J = J_{\max} \text{ at } y = b_T(x),$$
$$J = J_0 \text{ at } y = 0.$$

For the main section there is only one boundary condition, namely, the value of J(T) at the outer edge of the boundary layer.

As the initial condition, we assumed a uniform distribution of J along the y axis for each x_1 of the section where the combustion finished, since if it is supposed that the gas-air mixture is ignited, then at any distance from the exit section of the burner, the temperature is equilibrated in the transverse section and will be equal to the theoretical value.

As has already been noted above, the quantities U_j, V_j, J_{j-1}, J_{j-2} are nonlinearly dependent on y at a fixed value of x, while the term for heat generation of the chemical reaction qW is a function of J. Conversion of these functions during integration of the differential equations of the system was carried out by nonlinearity units.

The MN-7 analog computer can model differential equations containing up to four nonlinear relations which are functions of a single variable or of a product, while in the differential equations of system (II) there are five nonlinear relations and one product. Therefore, an attachment was devised to allow the machine to include an additional nonlinearity unit; in addition, the function V_j was approximated with the aid of two amplifiers.

The problem under examination was solved according to the block diagrams of Fig. 1, separately for the initial and main thermal sections, in the following sequence. The position of the initial section, in which the combustion process finishes, is not known. Therefore the distribution of excess enthalpy flux (or temperature) in the main section was determined by a successive approximation method, using as boundary condition values of J on the axis at the end of the initial section. This method was as follows. We assign arbitrarily the position x_1 of the section with uniform distribution of J_{max}, equal to the theoretical excess enthalpy. Taking the step equal to 0.025 m, we find the excess enthalpy flux distribution in the section located 0.025 m closer to the burner, by integrating the first equation of system (II) and assuming $J_{in-1} = J_{in}$. Integration was carried out according to the above block diagram, the value of the excess enthalpy flux on the axis of the main section being determined by trial and error (varying the value of J on the axis), using the other boundary condition—values of J at the outer edge of the boundary layer, i.e., the value of J on the axis was chosen in such a way that the other end of the curve of integration reached the assigned value of J at the outer edge.

Having determined $J_1 = f(y)$, we found the excess enthalpy flux distribution in the next section $x_1 = \Delta h$, and so on, up to x = 0.236 m (the end of the initial thermal section). If the value of J in the section x = = 0.236 m on the jet axis is equal to J_0, then the position of the section with uniform distribution of J_{max} has been assumed correctly; if not, then another position of the initial section with $J = J_{max}$ is assigned.

The distribution of excess enthalpy flux in the boundary layer of the initial thermal section is obtained directly, since the value of J at the internal and external boundaries is assigned. As the initial condition we took the distribution J = J(y) at the end of the initial thermal section, which was obtained from the solution of the main section.

To confirm the validity of the results of solution of system (II) and of the correctness of choice of the value of the coefficient ε_{Tj}, the equations of system (II) were also solved without thermal sources. Comparison of the results of these solutions with the known curves of distribution of T along the y axis confirmed the validity of the solutions obtained.

It is seen from Fig. 2 that the combustion process occurs only in a small part of the section, 10–15 mm, maximum heat release of the internal sources being observed not at the maximum temperature, but somewhat below this, approximately at T = 1473 °K. It should be noted that the graph of distribution of the longitudinal component of the effective velocity has a sharply pronounced bulge.

The curves presented in Fig. 3 are of considerable interest. It is characteristic that the curves of distribution of temperature along the axis of the jet, as well as of the longitudinal component of the effective velocity, have a less steep rise than in the transverse section.

The graphs of temperature distribution in transverse sections of the accompanying flows, and also the general configuration of the flame according to maximum heat release are given in Fig. 4.

NOTATION

u and v are the longitudinal and transverse components of the effective velocity; γ is the specific weight; ρ is the density; c_p is the specific heat at constant pressure; ΔT, ΔC are the excess temperature and concentration; ε_ν and ε_T are the dynamic and thermal coefficients of turbulent transfer; R, E are the gas constant and activation energy; q is the heat generation of chemical reactions; W is the rate of chemical reaction; $C^N_{CH_4}$, $C^N_{O_2}$, $C^0_{CH_4}$, $C^0_{O_2}$ are the concentrations of methane and of oxygen, variable and initial, referred to normal conditions; $2b_0$ is the width of the jet; x^d_{in}, x^T_{in} are the lengths of initial dynamic and thermal sections; b_V, b_T is the width of dynamic and thermal boundary layers; α is the air-fuel ratio; i is the enthalpy.

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1 March 1966

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